Fermi acceleration with memory-dependent excitation

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**ABSTRACT**

Some scaling properties for a classical particle confined to bounce between two walls, where one wall is fixed and the other one moves in time according to a random signal with a memory length are studied. We have considered two different kinds of collisions of the particle with the moving wall namely: (i) elastic and (ii) inelastic. The dynamics of the model is described in terms of a two-dimensional nonlinear mapping. For the case of elastic collisions, we show that the memory of the stochastic signal affects directly the behaviour of the average velocity of the particle. It then exhibits different slopes for the average velocity at different stages of the series with $\beta \approx 3/4$ for a short time, $\beta \approx 1$ for the average stage and $\beta \approx 1/2$ for a long time, as predicted by the Central Limit Theorem, therefore leading to the Fermi acceleration. The situation where inelastic collisions are taken into account yields a more drastic change, particularly suppressing the Fermi acceleration.

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1. Introduction

The phenomenon of unlimited energy growth for a classical particle generated via collisions with a time moving and heavy boundary is known in the literature as the Fermi acceleration (FA). The subject has been extensively studied in the past years and has applications in many different fields of science including plasma physics [1], astrophysics [2,3], atomic physics [4], optics [5–7] and the well-known time-dependent billiard problems [8]. The original idea was proposed by Enrico Fermi [9] as an attempt to explain the origin of the large energies of cosmic particles. After that, many different systems and models including their variants were proposed to describe and better understand the phenomenon.

The most important question that must be addressed is whether the FA can happen from the nonlinear dynamics itself in the absence of any random assumption. The answer for this question is not unique and depends on the properties of individual models. For one-dimensional systems, it is well known that, depending on the combination of both control parameters and initial conditions, the bouncer model (a classical particle falling in a constant gravitational field and bouncing elastically from a periodically moving platform) can indeed exhibit FA [10]. Such behaviour however is not observed for the one-dimensional Fermi–Ulam model (FUM), which consists of a classical particle of mass $m$ confined to bounce between two walls where one of them is fixed and the other one moves periodically in time [11]. There is also a recent result showing that, under a perturbation of the type crank-connecting rod scheme [12] and for specific ranges of control parameters (discontinuities on the moving wall velocity), the FUM exhibits FA.

The introduction of random perturbation to the wall leads to a drastic change on the dynamics. Since the seminal paper of Hammersley [13], it has been known that the average energy of the particle grows with function of both time and number of collisions with the wall when a random perturbation at each impact with the wall is introduced. This result was also confirmed for a stochastic version of the FUM under the framework of random shift perturbation [14,15].

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The situation for two-dimensional systems is rather different and depends basically on the geometry of the boundary. This is because the phase portrait of the static version of the model strongly depends on the shape of the boundary. Particularly, there is a well-known conjecture in the literature \[16\], the so-called LRA conjecture. It says that FA in time-dependent billiards should be observed if they possess chaotic component in the static case. Results that corroborate to the validity of this conjecture include the time varying circular billiard \[17\], the concentric case of the annular billiard \[18\] and the elliptic case \[19\]. A specific time perturbation in an elliptic billiard model was recently introduced by Lenz et al. \[20\]. It therefore leads the system to exhibit a tunable Fermi acceleration. The mechanism that accelerates the particle to unlimited energy is repeated crossings of the separatrix region by the particle's trajectory. It was discussed by Gelfreich and Turaev \[21\] a procedure at which a classical particle might acquire unlimited energy growth by using theory of Hamiltonian dynamics.

In this paper we revisit the one-dimensional Fermi–Ulam accelerator model to understand and describe the dynamics of the model, particularly the behaviour of the average velocity and hence is kinetic energy, when one of the walls moves randomly in time. However the procedure to generate the random series for the wall does indeed has a memory in time. We therefore consider such a memory in the description of the FA which exhibits different behaviour at different stages in time. We also consider the introduction of inelastic collisions of the particle with the randomly moving wall and study the influences of such a fractional loss of energy on the dynamics of the model. We confirm that inelastic collisions suppress FA in the dynamics of the model even for the stochastic signal with memory.

This paper is organised as follows. In Section 2 we discuss the model and in Section 3 the procedure used to construct the memory-dependent random time series is presented. We describe the behaviour of the average velocity as function of the number of impacts with the moving wall as well as function of the memory of the stochastic signal in Section 4. Section 5 is devoted to discussing the model and the behaviour of the average velocity as function of the restitution coefficient and the amplitude of oscillation of the moving wall. Our concluding remarks are drawn in Section 6.

2. The computational model

The present model abstracts the one-dimensional motion of a classical particle with mass \(m\), bouncing between two rigid walls as illustrated in Fig. 1. One of them is assumed to be fixed and the other one randomly moves with amplitude \(\varepsilon\), which is necessarily negligible regarding the simplifications adopted in the present FUM.

It shall be stressed here that \(\varepsilon\) must be sufficiently small if compared to the mean distance \(L\) between both walls in order to turn improbable that more than one collision with the moving wall may occur before the particle collides the fixed wall.

We consider that the collision of the particle with the moving wall is inelastic, which leads the particle to suffer a fractional loss of energy upon each collision with the moving wall. The inelastic behaviour is controlled by a prefixed restitution coefficient \(\alpha\) (\(0 \leq \alpha \leq 1\)), where \(\alpha = 1\) corresponds to the ideal case of elastic collisions, and for the other extreme i.e. \(\alpha = 0\) yields the trivial case of a sticky solution after the first collision of the particle with the moving wall.

The boundary conditions are then \(x = 0\), to represent the effective position of the moving wall, and \(x = L\), to represent the fixed wall. Since the moving wall movement is oscillatory, even with randomly drifting frequencies, the particle's velocity changes immediately after a collision with the moving wall. Moreover, the fixed wall works like a pure reflector, while the moving wall is a perturbing reflector. Such a perturbation function is the accelerator model properly said. Furthermore, velocities are computed after a collision with the moving wall, so that they are ever positive travelling from the moving wall to the fixed one.

Consistently with the previous assumption that just one collision might occur with the moving wall before the particle reaching the other wall, non-positive velocities are forbidden because they are equivalent to the particle travelling beyond the wall. In order to avoid such a problem, if after the collision the particle has a negative velocity, we artificially inject it back with the same modulus of velocity. Such a procedure is perfectly reproduced with the use of a modulus function.

Under the above considerations, we have

\[
T : \begin{cases} 
V_{n+1} = |\alpha V_n - (1 + \alpha)\varepsilon Z(t_{n+1})|, \\
t_{n+1} = t_n + \frac{2L}{V_n}
\end{cases}
\]

(1)

where the term \(\varepsilon Z(t)\) is the velocity of the moving wall at the time \(t\), with the velocity \(Z(t)\) normalized in the \([-1, 1]\)-range so that \(\sup Z(t) = 1\) and \(\inf Z(t) = -1\). The index \(n\) corresponds to the \(n\)th collision of the particle with the moving wall. The term \(2L/V_n\) furnishes the elapsed time between one hit and another one. From now on we assume as fixed the parameter \(L = 1\).

Our original contribution to this paper is therefore to consider that the moving wall is driven by a too simple memory-sensitive excitation, whose memory length is used as an input parameter of the collision simulations.

The simulation outcome is a time series of snapshots \((V_n, t_n)\) of the particle phase state when it impacts the moving wall. The computations are performed by means of a discrete mapping \(T : (V_n, t_n) \rightarrow (V_{n+1}, t_{n+1})\) given by Eq. (1).

3. Memory-sensitive excitation model

In order to check the Fermi–Ulam accelerator in scenarios other than in which the moving wall is excited by sinusoidal or even uncorrelated random force \(Z(t)\), we introduce a memory-dependent excitation model, which constraints the new
value for \( Z_n = Z(t_n) \) from the knowledge of the previous values \( Z_{n-1} = Z(t_{n-1}), Z_{n-2} = Z(t_{n-2}), \ldots, Z_{n-M} = Z(t_{n-M}) \), where \( M \) is the number of previous outcomes if \( n \geq M \). The warm-up sequence is then \( Z_0, Z_1, \ldots, Z_{n-1} \) if \( n < M \), since the number of past terms did not reach the maximum history size \( M \).

One possible approach to simulate memory-dependent stochastic processes is the accepting–rejecting method directed by a grammar so that the collection of the possible pseudo-random integers is the alphabet of input terminals. Thus, the next pseudo-random term looked ahead is accepted only if it matches the expected class of terms (tokens) as ruled by the grammar. A text from a large book is one possible instance of a grammar-directed stochastic process. However, it might be possible to find more complex approaches if the pseudo-random sequence is structured by grammars different levels of the Chomsky classification [22–24]. The behaviour of the FUM with formal languages shall be studied in a future work.

Another interesting approach resides in formulating the following aging equation:

\[
Z_n = \begin{cases} 
F(\xi_n|Z_{n-1}, \ldots, Z_{n-M}), & n \geq M; \\
G(\xi_n|Z_{n-1}, Z_{n-2}, \ldots, Z_0), & n < M,
\end{cases}
\]

where \( F \) and \( G \) are both real functions on the random variable \( \xi_n \), taken at time-step \( t_n \), and on the previous terms \( Z_{n-1}, Z_{n-2}, \ldots \) onto the \([-1, +1]\)-range.

\( F \) is the steady-state function and \( G \) is the warm-up function, which are arbitrarily defined to map the anomalous excitation function \( Z(t) \). We have adopted for \( F \) and \( G \) the arithmetic mean of the previous terms and the random number \( \xi_n \) normalized to the \([-1, +1]\)-range:

\[
Z_n = \begin{cases} 
\xi_n + \frac{\lambda Z_{n-1}}{1 + \lambda}, & n > 0; \\
\xi_0, & n = 0,
\end{cases}
\]

where \( \xi_0 \) is the initial random number and \( \bar{Z}_{n-1} \) is a moving average time series:

\[
\bar{Z}_{n-1} = \begin{cases} 
\frac{1}{M} \sum_{j=1}^{M} Z_{n-j}, & n \geq M; \\
\frac{1}{n} \sum_{j=1}^{n} Z_{j-1}, & n < M.
\end{cases}
\]

The \( \lambda \)-factor (\( \lambda > 0 \)) is the persistence parameter which expresses how the average past does influence the future terms. If \( \lambda = 0 \) the stochastic variable \( Z_n \) would be entirely random, which would be the trivial case. On the other hand, large values for \( \lambda \) yield a persistent behaviour to \( Z(t) \) in which the random variable \( \xi \) will have too low influence on \( Z(t) \) after \( M \) steps. We have adopted \( \lambda = 1 \) in our experiments in order to get a balanced competition among the random and the past terms.

The memory length \( M \) is in some sense a smoothing parameter and its effect might be in fact interpreted as a convolution of the previous random outputs with the top-hat function, whose length is roughly proportional to \( M \).

If the current solution is too far away from the initial condition, so that the number of steps is large enough to fairly approximate to the continuum, we can rewrite Eq. (3) as in the following non-computable [23–25] functional equation

\[
Z(t) = \frac{\xi(t)}{1 + \lambda} + \frac{\lambda}{1 + \lambda} \int_{t-\tau}^{t} \frac{1}{\tau} Z(u)du
\]

where \( \tau \propto M \), and \( \xi(t) \) is a noise term.

The functional term in the RHS of Eq. (5) is an integral interpolant whose kernel is the top-hat function with width equals to \( \tau \), and centered at \( t - \tau / 2 \). As higher is the value for \( \tau \) as much smoother is the signal provided by the functional. On the other hand, as smaller is \( \tau \) the steeper is the signal contribution from the functional. In the particular case, as \( \tau \to 0 \), Eq. (5) becomes equivalent to the identity \( Z(t) = \xi(t) \).
In order to illustrate how the memory-sensitive excitation \( M > 0 \) differs from the memoryless case \( M = 0 \), we performed two ensembles of random walk simulations, whose steps are given by \( Z_n \) from Eq. (3). Both ensembles comprise 64 simulations with 512 steps long. The persistence parameter is \( \lambda = 0.5 \) for both computer experiments. The first ensemble is the trivial case in which no memory is recalled on the choice of a new random step, which is illustrated in Fig. 2. The second ensemble is the memory-sensitive case, with the same parameters of the previous case but using \( M = 64 \), as shown in Fig. 3.

4. The computational experiment

A natural observable in problems involving FA is the average velocity. It is then obtained by two distinct steps: (i) The first step consists in averaging the velocity over the orbit for a single initial condition. Therefore, it is defined as

\[
V_i(n, \varepsilon, \alpha, M) = \frac{1}{n + 1} \sum_{j=0}^{n} V_{j,i},
\]

where the index \( j \) refers to the \( j \)th iteration of the sample \( i \); (ii) The second step is to evaluate the average over an ensemble of \( B = 5000 \) different initial conditions

\[
\overline{V} = \frac{1}{B} \sum_{i=1}^{B} V_i.
\]

We consider now the case of elastic collisions, i.e., the case where \( \alpha = 1 \). The behaviour of the average velocity \( \overline{V} \) for different values of the memory length is shown in Fig. 4. We can see that the \( \overline{V} \) depends directly on the control parameter corresponding to the memory of the random signal \( M \). For \( M = 0 \), the average velocity grows with the slope of growth given by \( \beta_3 \cong 1/2 \). Such kind of behaviour is mostly expected because the distribution of random signal is basically of Gaussian type. For \( M > 1 \) and larger, the behaviour is different and it can be described by the combination of three parts:
(1) it starts growing with a slope of the order \( \beta_1 \approx 3/4 \) (for the control parameter used in our simulation no more than \( n_x \approx 100 \) collisions with the moving wall—this is better seen in Fig. 5(a)) and then bends towards a regime of fast growth given by a power law with exponent \( \beta_2 \approx 1 \). There is also a changeover from a fast growth marked by \( \beta_2 \approx 1 \) to a growth characterised by \( \beta_3 \approx 1/2 \). The characteristic number of collisions where the changeover happens is given by \( n_x \).

Based on the behaviour shown in Figs. 4 and 5(a), we can describe the behaviour of \( \overline{V} \), for the non-dissipative case as:

- The average velocity is given by

\[
\overline{V} \propto \begin{cases} 
  n^{\beta_1} & \text{for } n \ll n_x \\
  n^{\beta_2} & \text{for } n_x \ll n \ll n_x \\
  n^{\beta_3} & \text{for } n \gg n_x,
\end{cases}
\]  

(8)

where \( \beta_i, i = 1, 2, 3 \) are critical exponents.
- The crossover iteration number \( n_x \) is described in terms of

\[
n_x \propto M^{z_2},
\]  

(9)

with \( z_2 \) being also a critical exponent.
Fig. 6. Plot of \(n_x \times M\). A power law fit gives \(z_2 = 1.73(4)\).

Fig. 7. (a) Behaviour of \(\bar{V}\) as a function of \(n\) for different values of the memory length \(M\). The control parameters used in the construction of the figure were \(\alpha = 0.99\) and \(\varepsilon = 10^{-2}\). (b) Collapse of the curves shown in (a) onto a single and universal curve.

There is also a critical exponent \(z_1\) which, for the control parameters used in our simulations, is difficult to obtain. Such a difficulty is mainly because the changeover from the initial growth characterised by \(\beta_1\) to the second and fast growth given by \(\beta_2\) occurs in a small range of \(n\), thus leading to \(n_{c1} < 100\).

After extensive simulations, we have obtained the following values for the critical exponents: \(\beta_1 = 0.743(6) \approx 3/4\), \(\beta_2 = 0.994(3) \approx 1\) and \(\beta_3 = 0.501(7) \approx 1/2\). The slopes of each growth curve are illustrated in Fig. 5(a). The critical exponent \(z_2\) emerges from a power law fitting via a plot of \(n_{x2} \times M\) where \(n_{x2}\) is obtained by a crossing of the second and fast regime of growth (\(\beta_2\)) with the regime of growth given by \(\beta_3\). It is shown in Fig. 6, the behaviour of \(n_{x2} \times M\). A power law fit furnishes us that \(z_2 = 1.73(4)\).

The behaviour shown in Fig. 5(a) and the characterisation of the critical exponents allow us to obtain, after a suitable rescaling of variables, a single and universal curve of \(\bar{V}\), as it is shown in Fig. 5(b). Basically, the important information that such behaviour brings is that the memory of the stochastic signal affects the regime of growth of the average velocity for small and average number of collisions with the moving wall. However, for a sufficient large number of collisions, the Central Limit Theorem applies and we obtain that the regime of growth is given with a critical exponent \(\beta_3 \approx 1/2\).

5. Consequences of dissipation on the dynamics

In this section we shall discuss the consequences of the introduction of a fractional loss of energy upon collision of the particle with the randomly moving wall. It is important to know first the influence of the memory \(M\) on the average velocity for a fixed restitution coefficient \(\alpha\). We have assumed as fixed the values \(\alpha = 0.99\) and \(\varepsilon = 10^{-2}\). It is then shown in Fig. 7(a) the behaviour of \(\bar{V} \times n\) for different values of \(M\). We can see that the curves start growing at small \(n\) and then they bend towards a regime of saturation. The changeover from growth to the saturation is marked by a typical crossover number \(n_s\).

The slope of growth as well as the regime of saturation depends on \(M\). The regime of growth is characterised by \(\bar{V} \propto n^\beta\) with \(\bar{\beta} = 0.749(5)\) for the range of \(M \in [20, 1000]\). The critical exponent \(\beta\) experiences a growth from \(\beta = 0.5\) up to \(\beta = 3/4\) for the range of \(M \in [0, 20]\). The regime of saturation is therefore described by \(\bar{V}_{\text{sat}} \propto M^\gamma\). The critical exponent \(\gamma\) is obtained by a power law fitting of \(\bar{V}_{\text{sat}} \times M\) as shown in Fig. 8. We can see that, after a regime of transition of small values of \(M\) say, \(M \leq 10\) there is no power law fitting the data. However, for \(M > 10\), a power law gives us that \(\gamma = -0.985(4) \approx -1\).

After a suitable change on the variables, we can see that all curves collapse onto a single and universal plot, as shown in Fig. 7(b). This result allow us to conclude that, any choice of \(M\) is feasible for the study of ranges of \(\alpha\) and ranges of \(\varepsilon\). We then keep, for simplicity and without losing generality, as fixed now the value of \(M = 50\).
Fig. 8. Behaviour of $V_{\text{sat}}$ as a function of $M$. The control parameters used in the construction of the figure were $\alpha = 0.99$ and $\varepsilon = 10^{-2}$. A power law fitting furnishes, for $M > 10$ that $\gamma = -0.985(4)$.

Let us now discuss the dynamics of $V$ as function of both $\alpha$ and $\varepsilon$. We note however that, for different values of $\varepsilon$, the average velocity curves start to grow with and then they bend towards a regime of saturation. However, different values of $\varepsilon$ lead the curves to start growing at different positions. We noted that an appropriate change of variables $n \rightarrow n \varepsilon$ coalesce all the curves at small $n$, as can be seen in Fig. 9(a). Other important information is that $\alpha$ is not a good scaling variable. Instead of using it, we shall consider $\alpha \rightarrow (1 - \alpha)$ because it brings the criticality to the origin of the coordinate system. Considering these variables, we can propose the following scaling hypotheses for $V$:

- That the average velocity $V$ grows according to
  $$V \propto (n \varepsilon)^\beta \quad \text{for } n \ll n_x,$$
  where $\beta$ is a critical exponent.
- For sufficient long time, the average velocity is given by
  $$V_{\text{sat}} \propto (1 - \alpha)^{\gamma_1 \varepsilon^{\gamma_2}} \quad \text{for } n \gg n_x,$$
  where both $\gamma_1$ and $\gamma_2$ are critical exponents.
- The number of collisions that marks the change from growth to the saturation is written as
  $$n_x \varepsilon \propto (1 - \alpha)^{z_1 \varepsilon^{z_2}},$$
  where $z_1$ and $z_2$ are the so called dynamical exponents.

These scaling hypotheses allow us to formally describe the average velocity as a function of the type
$$V(n \varepsilon, \varepsilon, (1 - \alpha)) = l V_1((n \varepsilon)^{-b/a}, (n \varepsilon)^{-c/a}(1 - \alpha)),$$
where $l$ is a scaling factor and $a$, $b$ and $c$ are scaling exponents that must be related to the critical exponents $\gamma_i, z_i$ with $i = 1, 2$ and $\beta$.

Since $l$ is a scaling factor, we can chose it such that $l = (n \varepsilon)^{-1/a}$, yielding
$$V(n \varepsilon, \varepsilon, (1 - \alpha)) = (n \varepsilon)^{-1/a}V_1((n \varepsilon)^{-b/a}, (n \varepsilon)^{-c/a}(1 - \alpha)),$$
where $V_1((n \varepsilon)^{-b/a}, (n \varepsilon)^{-c/a}(1 - \alpha)) = V_1(1, (n \varepsilon)^{-b/a}, (n \varepsilon)^{-c/a}(1 - \alpha))$ is assumed to be constant for $n \ll n_x$. Comparing Eq. (14) with Eq. (10) yields $\beta = -1/a$. After extensive numerical simulations, we obtain that $\beta = 0.908(8)$.
Choosing now $l = \varepsilon^{-1/b}$, we have that

$$\bar{V}(n\varepsilon, \varepsilon, (1 - \alpha)) = \varepsilon^{-1/b}V_2(\varepsilon^{-a/b}n\varepsilon, \varepsilon^{-c/b}(1 - \alpha)),$$

where $V_2(\varepsilon^{-a/b}n\varepsilon, \varepsilon^{-c/b}(1 - \alpha)) = \bar{V}(\varepsilon^{-a/b}n\varepsilon, 1, \varepsilon^{-c/b}(1 - \alpha))$ is assumed to be constant for $n \gg n_\varepsilon$. Comparing Eqs. (15) and (11) we obtain $\gamma_2 = -1/b = 1.003(2)$. Using the two scaling factors and a comparison with Eq. (12) we are able to obtain that $z_2 = \gamma_2/\beta$. Evaluating such an expression, we obtain that $z_2 = 1.105(7)$ which is quite close to the value obtained in the Fig. 10(c).

Let us now chose $l = (1 - \alpha)^{-1/c}$, thus leading to

$$\bar{V}(n\varepsilon, \varepsilon, (1 - \alpha)) = (1 - \alpha)^{-1/c}V_3((1 - \alpha)^{-a/c}n\varepsilon, (1 - \alpha)^{-b/c}\varepsilon),$$

where $V_3((1 - \alpha)^{-a/c}n\varepsilon, (1 - \alpha)^{-b/c}\varepsilon) = \bar{V}((1 - \alpha)^{-a/c}n\varepsilon, (1 - \alpha)^{-b/c}\varepsilon, 1)$ is assumed to be constant for $n \gg n_\varepsilon$. Comparing Eqs. (16) and (11) we obtain $z_1 = a/c$. A power law shown in Fig. 10(a) shows that $z_1 = -0.91(2)$. Using now the expressions obtained for the scaling factor and a comparison with Eq. (12), we obtain $z_1 = -0.91(3)$ which is in good agreement with the result shown in Fig. 10(a).

To confirm that the scaling hypotheses proposed in Eqs. (10)-(12) are correct, after a suitable rescaling in the axis, all the curves are merged together onto a single and universal curve, as shown in Fig. 9(b).

We can see however, after the introduction of dissipation via a restitution coefficient, the velocity of the particle becomes then limited to a finite range, therefore suppressing the Fermi acceleration. We can conclude that, even for stochastic dynamics generated in such a way that exhibits a memory length, the Fermi acceleration is destroyed thus leading the average velocity of the particle to reach a constant value.

6. Conclusions

In summary we have studied some dynamical properties of a simplified version of the one-dimensional Fermi accelerator model under stochastic perturbation. We have considered two regimes for the dynamics: (1) non-dissipative regime and (2) dissipative dynamics. For case (1) we have shown that the memory affects the regime of growth for the average velocity. The particle thus has three regimes of growth that are marked by three different growing exponents. For short time, the average velocity grows with exponent $\gamma_1 \approx 3/4$. For intermediate time, the slope of growth is $\gamma_2 \approx 1$ while for long time, and as the Central Limit Theorem predicts, the slope of growth is $\beta_3 \approx 1/2$. For the dissipative case, the dynamics is drastic affected. Particularly, the Fermi acceleration is suppressed and the particle reaches, after a regime of growth, a plateau of constant velocity. We have obtained critical exponents that describe the behaviour of the average velocity and shown that such a behaviour exhibits patterns of universality. The most important conclusion is that the inelastic collisions are an efficient mechanism to suppress the Fermi acceleration, as previously discussed in Ref. [26].

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